# B.3 Spatial Autocorrelation

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## B.3.1 Introduction

In this chapter we review the concept of spatial autocorrelation and its attributes. Our purpose is to outline the various formulations and measures of spatial autocorrelation and to point out how the concept helps assess the spatial nature of georeferenced data. For a fuller treatment of the subject, a number of texts, written at various junctures in the development of the concept and at differing levels of mathematical sophistication, spell out many of the details not discussed here (Cliff and Ord 1973, 1981; Miron 1984; Upton and Fingleton 1985; Goodchild 1986; Odland 1988; Anselin 1988; Haining 1990a; Legendre 1993; Dubin 1998; Griffith 1987, 1988, 2003). In addition, and as background to this chapter, Haining's contribution in this volume (see Chapter B.1) gives a clear view of the nature of georeferenced data. Our goal is to briefly describe the literature on this subject so that the spatial autocorrelation concept is accessible to those who (i) are new to dealing with georeferenced data in a research framework or (ii) have worked with georeferenced data previously but without explicit knowledge of how the concept can be beneficial to them in their research. We are constrained by space and, as a result, our plan is to be short on explanations but identify key literature where the reader will find further details.

After defining and briefly giving the background for the concept of spatial autocorrelation in this section, we explain the concept's attributes and uses in Section B.3.2. In the next section, we discuss the matrices that must be created in order to assess most measures of the spatial autocorrelation concept. We outline the various spatial autocorrelation formulations in Section B.3.4. This is followed in Section B.3.5 with a short discussion of the problems in applying the concept in research situations. Finally, Section B.3.6 provides a brief description of available spatial autocorrelation software. The reference list can serve as a guide to the literature in this area.

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### *Definitions*

The simplest definition of the spatial autocorrelation concept is that it represents the relationship between nearby spatial units, as seen on maps, where each unit is coded with a realization of a single variable. Adding more detail and conciseness, as Hubert et al. (1981, p.224) put it:

> '*Given a set S containing n geographical units, spatial autocorrelation refers to the relationship between some variable observed in each of the n localities and a measure of geographical proximity defined for all n*(*n*–1) *pairs chosen from S.*'

If a matrix $\boldsymbol{Y}$ represents all of the $(n^2–n)$ associations between all realizations of the $\boldsymbol{Y}$ variable in region $\mathcal{R}$ and $\boldsymbol{W}$ represents all of the $(w^2–w)$ associations of the spatial units to each other in region $\mathcal{R}$, irrespective of $\boldsymbol{Y}$, then the degree to which the two matrices are positively (negatively) correlated is the degree of positive (negative) spatial autocorrelation. Thus, if it is assumed that neighboring spatial units are associated and so are represented in the $\boldsymbol{W}$ matrix as high positive numbers and low numbers or zero for all others, and the $\boldsymbol{Y}$ matrix has high values in spatial units neighboring other high values, then the two matrices are similar in structure with the result that positive spatial autocorrelation exists.

### *Development of the concept*

The spatial autocorrelation concept was bred at the University of Washington in the late 1950s, principally by Michael F. Dacey, mainly in the presence of William L. Garrison and Edward Ullman, two geographers very much influenced by the central place work of the 1930s German economic geographer Walter Christaller. Earlier, an extensive literature had been developed on the principal of nearness, that is, the strong effect that nearby areas have on each other versus the relatively weak influence of areas further away (for example, Ravenstein 1885; von Thünen 1826; Zipf 1949) with the implication that near spatial units are similar to one another. This notion is best summarized by Tobler's First Law, '*Everything is related to everything else, but near things are more related than distant things*' (Tobler 1970, p.234). The roots of the idea go back to Galton, Pearson, Student, and Fisher. Until 1964, in the social science and statistics literature, spatial autocorrelation had been called 'spatial dependence,' 'spatial association,' 'spatial interaction,' 'spatial interdependence,' among other terms. In geography, the modern meaning of the term 'spatial autocorrelation' was first mentioned by Garrison in or before 1960 (Thomas 1960, in Berry and Marble 1968), and first developed in a statistical framework by Cliff and Ord (1969).

Three statisticians laid out the mathematical characteristics of spatial autocorrelation, although they used the term *contiguity ratio* to describe their work. Moran (1948), Krishna-Iyer (1949), and Geary (1954) developed join count statis-

tics based on the probability that joined spatial units were of the same nominal type (black or white) more than chance would have it. Their work was extended to take interval data into account. Geary, in particular, made the point that the mapped residuals from an ordinary least squares regression analysis must display the characteristic of independence. Dacey further explicated join count statistics, extending the number of colors studied from two to *k*, and clearly showed the link between using nominal and interval data (Dacey 1965). Also, Dacey recognized the possible effect of the shapes, sizes, and boundaries of regions (topological invariance) on the results of analyses that used georeferenced data (Dacey 1965).

In the field of geostatistics, Matheron (1963) had already developed in considerable detail the mathematics that accompanies the assumption of *intrinsic stationarity*, the notion that inherently characteristic of spatial distributions is a distance effect. Without using the term spatial autocorrelation, the *correlogram* (the inverse of the semivariogram), was invented to represent intrinsic stationarity, the declining similarity of variable values assumed to exist among spatial units as distance increased from each other.

The monograph *Spatial Autocorrelation* by Cliff and Ord (1973) sheds light on the problem of model mis-specification owing to spatial autocorrelation and demonstrated statistically how one can test residuals of regression analysis for spatial randomness by using spatial autocorrelation statistics. Models that require traditional statistics for their evaluation are mis-specified if they do not take spatial autocorrelation into account. The moments of Moran's distribution, called Moran's *I*, were fully developed by Cliff and Ord (1973, 1981) under varying sampling assumptions.

## B.3.2 Attributes and uses of the concept of spatial autocorrelation

The following list gives some idea of the range of uses for the concept and for the formulas created to measure the degree of spatial autocorrelation in modeling situations. The list should convince all of those who deal with georeferenced data that an explicit recognition of the concept is basic to any spatial analysis.

- *A test on model mis-specification.* Properly specified models that call for normally distributed residuals also require that residuals map onto the study region in such a way that one cannot detect any association between nearby spatial units. Proper specification requires that any spatial association is subsumed within the model proper. The most used, and statistically most powerful, test for detecting the spatial independence of residuals is that of the spatial autocorrelation statistic, Moran's *I* (Cliff and Ord 1972, 1981; Anselin 1988).
- *A measure of the strength of the spatial effects on any variable.* A thorough understanding of the effects of regressor variables on a dependent variable re-

quires that any spatial effects on both dependent and independent variables be quantified. Spatial autocorrelation coefficients in regression models help us to understand the strength of spatial effects (Haining 1990b; Anselin and Rey 1991).

- *A test on assumptions of spatial stationarity and spatial heterogeneity.* Before engaging in many types of spatial analysis, it is necessary to make the assumption that spatial stationarity exists. There are many definitions of spatial stationarity; most common is that the mean and variance of a variable under consideration do not vary appreciably from subregion to subregion in the study region. Spatial autocorrelation measures allow for tests on hypotheses of no spatial differences in distribution parameters such as the mean and variance (Haining 1977; Leung 2000).
- *A means of identifying spatial clusters.* Spatial clustering algorithms are dependent on the conjecture that there is spatial autocorrelation among some nearby values of one or more variables of interest. The basis of clustering computer routines such as ClusterSeer, StatScan, and AMOEBA is the concept of spatial autocorrelation (Aldstadt and Getis 2006).
- *A means of identifying the role that distance decay or spatial interaction might have on any spatial autoregressive model.* Measures of spatial autocorrelation can identify the parameters of spatial decay (for example, the parameters of a negative exponential model) or the parameters of spatial interaction models (Fotheringham 1981).
- *A way to understand the influence that the geometry of spatial units has on a variable.* Measures of spatial autocorrelation will change in certain known ways when the configuration of spatial units changes. These measures are ideal for understanding the role that spatial scale might have on relationships among georeferenced variables (Arbia 1989; Wong 1997). Also see Okabe et al. (2006) on network configurations and spatial autocorrelation.
- *A test on hypotheses about spatial relationships.* Spatial autocorrelation statistics are usually designed to test the null hypothesis that there is no relationship among realizations of a single variable, but the tests may be extended to consider spatial relations between variables (Wartenberg 1985).
- *A means of weighing the importance of temporal effects.* A series of measures of spatial autocorrelation taken over time sheds light on temporal effects (Rey and Janikas 2006).
- *A focus on a single spatial unit's effect on other units and vice versa.* The local view of spatial autocorrelation (see below) allows for focused tests where a particular spatial unit is the focus (Ord and Getis 1995; Anselin 1995; Sokal et al. 1998).
- *A means of identifying outliers, both spatial and non-spatial.* Certain statistical and graphical routines allow for the exact identification of units that unduly influence spatial effects (Anselin 1995).
- *A help in designing an appropriate spatial sample.* If the goal is to avoid, as much as possible, spatial autocorrelation in the sample, then a reasonable sam-

ple design would benefit from a study of spatial autocorrelation in the region where the sample is to be selected (Fortin et al. 1989; Legendre et al. 2002; Griffith 2005).

The list can be expanded, but suffice it to say here are many characteristics of spatial autocorrelation that add depth and understanding to any spatial analysis.

## B.3.3 Representation of spatial autocorrelation

Since the types of studies in which the concept of spatial autocorrelation is used vary considerably, many methods and techniques of analysis have been created for special purposes. The following simple representation of spatial autocorrelation is the key to the proper choice of measure or test (Hubert and Golledge 1981; Getis 1991).

### *The cross-product statistic*

$$\Gamma_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij} Y_{ij} \tag{B.3.1}$$

where $\Gamma$ is a measure of spatial autocorrelation for $n$ georeferenced observations. It is made up of $\boldsymbol{W}$, a matrix of values that represents the spatial relationships of each location $i$ to all other sites $j$. The $\boldsymbol{Y}$ matrix shows the non-spatial relationship of realizations of a variable $\boldsymbol{Y}$ at site $i$ with all other realizations at all other sites $j$. When $\boldsymbol{W}$, the spatial weights matrix, and $\boldsymbol{Y}$, the variable matrix have similar structures [for example, both have high values in the same $(i, j)$ cells in their respective matrices and low values in the same $(i, j)$ cells] one can say that there is a high degree of spatial autocorrelation. The correlation can be positive or negative depending on whether respective cells are similarly matched or oppositely matched. If realizations of $\boldsymbol{Y}$ are randomly placed in the spatial units, no matter how the spatial weights matrix is structured, the result will be a $\Gamma$ of zero, or no spatial autocorrelation. The same is true if the $\boldsymbol{W}$ matrix is based on random spatial associations and the $\boldsymbol{Y}$ happens to be spatially structured. Thus, it is clear that for any meaningful assessment of spatial autocorrelation the $\boldsymbol{W}$ matrix must be a careful representation of spatial structure and the $\boldsymbol{Y}$ matrix must represent a meaningful association between realizations of the $Y$ variable. Equation (B.3.1), as it is presented, is not a test of spatial autocorrelation, but only a measure. Tests on the existence of spatial autocorrelation, however, take on the same cross-product structure. In the next section, the structure of $\boldsymbol{W}$ matrices is discussed.

### The **W** matrix

The ***W*** matrix embodies our preconceived or derived understanding of spatial relationships. If we believe or if theory tells us that a particular spatial relationship is distance dependent, then the ***W*** matrix should reflect that supposition. For example, if it is assumed that a spatial relationship declines in strength as distance increases from any given site, then the ***W*** matrix will show that nearby areas are weighted more highly than sites that are far from one another. Various distance-decay formulations theorized or derived for such phenomena as travel behavior, economic interaction, or disease transmission would require the elements within the ***W*** matrix to reflect these effects. Thus, a typical ***W*** matrix might contain matrix elements (represented as lower case letters) of the form

$$W_{ij} = d_{ij}^{-\alpha} \quad \text{with } \alpha \geq 1. \tag{B.3.2}$$

Or, in words, the weight entered into cell $(i,j)$ is the inverse of distance $d$ between the two sites, $i$ and $j$, reduced by the exponent $\alpha$, where $\alpha$ is greater than one. The ***W*** matrix can represent distances other than those derived from Cartesian geometry. For example, friendship or cell phone networks may be distance related in sociological terms. A bevy of schemes have been created to attempt to fashion ***W*** (Getis and Aldstadt 2004). Some of the schemes are:

- Spatially contiguous neighbors (default for many studies),
- Inverse distances raised to some power (distance decline function),
- Lengths of shared borders divided by the perimeter (a geometric view),
- Bandwidth as the *n*th nearest neighbor distance (point density dependent),
- Ranked distances (non-Cartesian approach),
- All centroids within distance $d$ (density dependent),
- $n$ nearest neighbors (equal weighting of matrix entries),
- Bandwidth distance decay (required for geographically weighted regression),
- Gaussian distance decline (based on the square term),
- Derived spatial autocorrelation (based on observed spatial association).

Perhaps the most used ***W*** is the first in the list above. ***W*** is made up of ones for contiguous neighbors and zero for all others, whether the data are raster or vector. By convention, the *i*th observation is not considered a neighbor of itself. The contiguity ***W*** matrix is often row-standardized, that is, each row sum in the matrix is made to equal one, the individual $W_{ij}$ values are proportionally represented. Row-standardization of ***W*** in contiguity schemes is desirable so that each neighbor of a spatial unit is given equal weight and the sum of all $W_{ij}$ is equal to $n$. As we will later see, these characteristics enhance understanding of spatial autocorrelation measures and coefficients. Researchers should be aware, however, that row-

standardization may give too much weight to observations with few spatial links and not enough weight to observations having many contiguous neighbors (Tiefelsdorf et al. 1998).

Those developing spatial models consider the spatial weights matrix to be one of the following three types of representations:

(i) a theoretical notion of spatial association, such as a distance decline function,
(ii) a geometric indicator of spatial nearness, such as the representation of contiguous spatial units,
(iii) some descriptive expression of the spatial association already existing within a set of data.

For viewpoint one, modelers argue that a $\boldsymbol{W}$ matrix is exogenous to any system and should be based on a pre-conceived matrix structure. A typical theoretical formulation for $\boldsymbol{W}$ would be based on a strict distance decline function such as shown in Eq. (B.3.2). Since little theory is available for the creation of these matrices, many researchers follow viewpoint two, that is, they resort to geometric $\boldsymbol{W}$ specifications, such as a contiguity matrix, reasoning that it is the nearest neighboring spatial units that bear most heavily on spatial association in a typical set of georeferenced data. Tiefelsdorf (2000) has created a system for coding these and other matrices based on geometric structure that goes well beyond simple contiguity matrices.

For viewpoint three, modelers allow study data to 'speak for themselves,' that is, they extract from the already existing data whatever spatial relationships appear to be the case and then create a $\boldsymbol{W}$ matrix from the observed spatial associations. As a result, models based on this type of endogenous specification have limited explanatory power, the limit being the reference region. Kooijman (1976) proposed to choose $\boldsymbol{W}$ in order to maximize Moran's coefficient (see next section). Reinforcing this view is Openshaw (1977), who selects that configuration of $\boldsymbol{W}$ which results in the optimal performance of the spatial model. Getis and Aldstadt (2004) construct $\boldsymbol{W}$ by using a local spatial autocorrelation statistic to generate the $W_{ij}$ from the data.

The nature of the variables being studied for spatial effects is the key to an appropriate $\boldsymbol{W}$. Variables that show a good deal of local spatial heterogeneity at the scale of analysis chosen would probably be more appropriately modeled by few links in $\boldsymbol{W}$, while a homogeneous or spatial trending variable would better be modeled by a $\boldsymbol{W}$ with many links. This implies that the scale characteristics of data are crucial elements in the creation of $\boldsymbol{W}$. As spatial units become large, spatial dependence between units tends to fall (Can 1996).

### *The **Y** matrix*

The non-spatial matrix, ***Y***, provides a view of how the realizations of a variable are associated with one another. ***Y*** represents the interaction of the elements $y_{ij}$. They may interact by an additive ($y_i + y_j$), multiplicative ($y_i\, y_j$), differencing ($y_i - y_j$), or division ($y_i / y_j$) process. A useful type of multiplicative matrix is the covariance matrix $(y_i - \bar{y})\,(y_j - \bar{y})$. All of these matrices can be scaled in order to serve a particular view of relationships within a variable. In the following section, we present the scaling of these processes for the creation of various views of spatial autocorrelation. In sum, the measures and tests for spatial autocorrelation differ by use and by the structure of their ***W*** and ***Y*** matrices.

## B.3.4 Spatial autocorrelation measures and tests

Spatial autocorrelation measures can be differentiated from tests on spatial autocorrelation by purpose, but both allow for the assessment of spatial effects in any analysis of georeferenced data. Moran's *I*, discussed below, is both the leading measure of and leading test on spatial autocorrelation, while, for example, the Kelejian-Robinson test on spatial autocorrelation is not used as a measure. Also, measures of spatial autocorrelation of the correlogram are not used as tests on spatial autocorrelation.

Spatial autocorrelation measures and tests can be differentiated by the scope or scale of analysis. Traditionally, they are separated into 'global' and 'local' categories. Global implies that all elements in the ***W*** and ***Y*** matrices taken together are brought to bear on an assessment of spatial autocorrelation, that is, all associations of spatial units one with another are included in any calculation of spatial autocorrelation. This results in one value for spatial autocorrelation for any one ***W*** and ***Y*** matrix taken together. Local measures are focused, that is, they usually assess the spatial autocorrelation associated with one particular spatial unit. Thus, only one row of the ***W*** and the matching row of the ***Y*** matrix reflect on the measure of spatial autocorrelation although all elements' interactions may be used as a scalar.

### *Global measures and tests*

*Gamma* ( $\Gamma$ ). As discussed earlier, this measure was used in our discussion as the basis on which all spatial autocorrelation measures and tests are structured. A test on the statistical significance of $\Gamma$ is made practical by randomizing *Y* values in a number of simulations. The observed $\Gamma$ can then be compared to the envelope created by the results of the simulations. Statistical significance implies that spatial autocorrelation exists.

*Join-count.* The purpose here is to identify for an exhaustive nominal classification of spatial units, such as for land use types – residential (*A*), industrial (*B*), commercial (*C*) – whether there are statistically significant numbers of spatially associated *AA, AB, AC, BB, BC*, and/or *CC* occurrences. In a system of spatial units, the expected number of *AA*, for example is a function of the type of test that is selected for identifying statistical significance. Here we use the free sampling test (Cliff and Ord 1981).

Given the probability $p_r$ that a spatial unit is a particular type of land use, and the number of units of that type is $n_r$, the expected number of joins of the same type is

$$E(J) = \tfrac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}\, p_r^2 . \tag{B.3.3}$$

For different types, the expectation is

$$E(J) = \sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}\, p_r\, p_s . \tag{B.3.4}$$

The $p$ values are usually estimated from the data ($n_r / n$). The $\boldsymbol{W}$ matrix is made up of ones and zeros representing joined spatial units (one) and non-joined spatial units (zero). There is a series of $\boldsymbol{Y}$ matrices, one for each test, where each is made up of ones and zeros representing specified types of associated spatial units (for example, *AB* is one and not *AB* is zero) and summarized as the probabilities of occurrence of *A* and *B* ($p_r$ and $p_s$). In order to perform tests on spatial autocorrelation, the variance must be known and the assumption invoked of an asymptotic normal distribution of the frequency of cells (see Cliff and Ord 1981 for details).

*Moran's I.* This statistic is structured as the Pearson product moment correlation coefficient. The crucial difference is that space is included by means of a $\boldsymbol{W}$ matrix and instead of finding the correlation between two variables, the goal is to find the correlation of one variable with itself vis-à-vis a spatial weights matrix. The $\boldsymbol{Y}$ is a covariance matrix, that is, Moran's *I* focuses on each observation as a difference from the mean of all observations. Set $\boldsymbol{W}$ to a preferred or required spatial weights matrix (any of those listed above), set $\boldsymbol{Y}$ equal to the auto-covariance $(y_i - \bar{y})\,(y_j - \bar{y})$, and scale the measure (invoking a Pearson limit structure) by multiplying by

$$\frac{n}{W}\left[\sum_{i=1}^{n} (y_i - \bar{y})^2\right] \tag{B.3.5}$$

where

$$W = \sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij} \tag{B.3.6}$$

and, by convention, $i$ is not to equal $j$ (no self association). We have then

$$I = \frac{n}{\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}} \quad \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij} (y_i - \bar{y}) - (y_j - \bar{y})}{\sum_{i=1}^{n} (y_i - \bar{y})^2} \quad i \neq j. \tag{B.3.7}$$

The expected value is $E(I) = -1/(n-1)$ and the variance is calculated somewhat differently under an assumption of randomness versus an assumption of normality. These two assumptions represent the supposed theoretical way the $Y$ values were produced under the hypothesis of randomly placed $Y$ values. Thus, Moran's $I$ is a test for spatial randomness; rejection of the null hypothesis implies with a certain degree of certainty (for example, 95 percent) that spatial autocorrelation exists. The randomness assumption ($R$) implies that the values of $y$ are realizations of a single uniformly distributed $Y$ variable (that is, a variable where all possible realizations are equally likely). The normal assumption means that each $y$ value is a randomly selected realization of a different normal distribution, one representing each spatial unit. It should be pointed out that a variation exists for Moran's $I$ when residuals of regression are being tested for spatial randomness. This is

$$I = \frac{n}{\sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}} \quad \frac{\boldsymbol{\varepsilon}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon}^{\mathrm{T}} \boldsymbol{\varepsilon}} \tag{B.3.8}$$

where $\boldsymbol{\varepsilon}$ is a vector of ordinary least squares residuals and $\boldsymbol{\varepsilon}^{\mathrm{T}}$ is the matrix transpose. The expected value and variance are a function of the number of independent variables in the system (Cliff and Ord 1972).

Moran's $I$ can be used in a wide variety of circumstances. As a global statistic, Moran's $I$ quickly indicates not only the existence of spatial autocorrelation (positive or negative) but also the degree of spatial autocorrelation If the variable of interest is the error term in a regression model, the question of model misspecification can be evaluated by applying Moran's $I$. In spatial econometrics, the test has power for testing residuals for many types of spatial autoregressive

models (Anselin 2006). Since Moran's $I$ is distributed normally, its value may be assessed by the $z$ values of the normal distribution. The statistic is flexible in that the $\boldsymbol{W}$ matrix may be of any form – it has no restrictions on the spatial system used. Of course, outliers in one or both of the $\boldsymbol{W}$ and $\boldsymbol{Y}$ matrices will yield meaningless results. The local version of Moran's $I$, discussed later, lends itself to spatial cluster identification and spatial filtering. A large literature has been developed to explore the properties of Moran's $I$. In addition to the basic references given in the first paragraph of this contribution, see Tiefelsdorf and Boots (1995, 1997); Hepple (1998).

*Geary's c*. The particular test employed for spatial autocorrelation is a function of the type of hypothesis required for the analysis. In the case of Moran's $I$, the null hypothesis was based on a covariance structure, that is, the expectation that related neighbors co-vary in no consistent way. For Geary's $c$, the null hypothesis is that related spatial units do not differ from one another. The implication of this hypothesis is that the expectation is that there is no consistency to the differences between neighbors; sometimes the differences are large and sometimes small. In this case, as for Moran's $I$, the $\boldsymbol{W}$ matrix is made up of any meaningful spatial relations between spatial units. The $\boldsymbol{Y}$ matrix is simply made up of the differences in the realizations of the variable $Y$ among all observations: $(y_i - y_j)^2$. A scale is included so that the resulting structure is normal, thereby lending Geary's $c$ to statistical tests. Thus, we have

$$c = \frac{(n-1)\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}\,(y_i - y_j)^2}{2\,W\sum_{i=1}^{n}(y_i - \bar{y})^2} \quad i \neq j. \tag{B.3.9}$$

Note that the scale results in an expected value of Geary's $c$ as one. In tests, values less than one indicate positive spatial autocorrelation (small differences) and values greater than one imply negative spatial autocorrelation (consistently large differences). Geary's $c$ is negatively related to Moran's $I$. Many of the references already given for Moran's spatial autocorrelation statistic contain references to Geary's measure.

*The variogram*. Central to the field of geostatistics is the semivariogram. Cressie (1993) provides a detailed treatment of the concept. Suffice is to say here that the semivariogram is a distribution of differences among spatially associated units and therefore is related to Geary's $c$. The major difference is that the semivariogram hypothesizes that the differences decline with distance from each other in a systematic way. Thus, the semivariogram describes a continuous view of differences, while Geary's statistic is relegated to one $\boldsymbol{W}$. A typical semivariogram has the shape of a positive exponential distribution, where close

distances display small differences and low variances, and far distances are not affected by distance effects in such a way that when all differences are taken together the value of the global variance obtains. The semivariogram has the form

$$\gamma\,(ad) = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}\;(y_i - y_j)^2 \qquad \text{(B.3.10)}$$

where there is a $\boldsymbol{W}$ for each constant distance $d$ controlled by an integer multiplier $a$. Thus, a particular constant distance (say, one kilometer) has a $\boldsymbol{W}$ for each $a$. But, the $\boldsymbol{W}$ matrices are constrained to contain ones and zeros. In effect, the $\boldsymbol{W}$ matrix identifies spatial units that are related at a particular distance ($ad$) or a particular band from each observation. The display of the spatial autocorrelation is called the correlogram, a function that decreases with distance until the *range* is reached. The *range* represents a distance where the global variance is unaffected by distance effects. The scale of the semivariogram, 1/2, is a recognition that there is double counting, the differences between $i$ and $j$ are the same as between $j$ and $i$. Cressie (1993) provides a comprehensive treatment of geostatistics, and Rosenberg et al. (1999) emphasizes the spatial autocorrelation aspects of the analysis.

*Ripley's K function.* As is true of the correlogram, Ripley's $K$ function (Ripley 1976; Besag 1977) represents a continuous set of spatial autocorrelation indicators. The $K$ function, unlike the measures discussed previously, emphasizes only location and not the other attributes of a random variable. So here we are restricted to point patterns based on the number of pairs of points found at a series of distances from each $i$th point. In this case, the object is to count all pairs of points at each distance. If there are more pairs of points than spatial random chance (spatial Poisson distribution) would have it, there is statistically significant clustering; fewer pairs of points implies a statistically significant dispersion of points, the opposite of clustering. The null hypothesis obtains when there are about as many pairs of points as one might find in a point distribution created by a random process. A random spatial process is called a homogeneous Poisson process over the study plane, that is, all sites within the area of study are equally likely to receive a point, and the siting of a point in no way bears on the siting of another point. The statistic is estimated in the following way

$$\hat{K}(d) = \frac{R}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{W_{ij}}{e_{ij}} \quad i \neq j\,. \qquad \text{(B.3.11)}$$

Within the study region of size $R$, for distance $d$ we count all of the pairs of points that are not larger than $d$ apart. Thus for a number of increasing distances the value of $K(d)$ will increase as more pairs are added to the total. The $\boldsymbol{W}$ matrix is made up of one for $(i, j)$ pairs within $d$ of one another and zero otherwise. As distance increases, the boundary of the region is more likely to be closer to a point $i$ than to any $j$ point. In that case, an edge correction $e_{ij}$ is invoked which assumes that any point outside of the boundary is unobserved but that the point process continues for at least a short distance beyond the boundary. Center a circle of radius $d_{ij}$ on $i$, and if the circle crosses the boundary, the proportion of the circumference of the circle that lies inside the study area replaces that particular pair count of one to a value greater than one, thus insuring consistency of the presumed point process. Of course, points close to the boundary but far from a neighbor distort any result. Further, by including the estimate of $\hat{K}(d)$ in the following formula, a significant improvement is made for recognizing spatial autocorrelation in a point process.

$$\hat{L}(d) = \sqrt{\frac{\hat{K}(d)}{\pi}}. \tag{B.3.12}$$

When this formula is used, the expectation based on the hypothesis of Poisson randomness becomes a positive straight line where $\hat{L}(d) = d$. Typically a series of Poisson random distributions is simulated, helping to create an envelope containing, say, 95 percent of possible point patterns under the hypothesis of randomness. An observed pattern whose $L(d)$ value falls outside of the envelope indicates the existent of positive (clustering) or negative (dispersion) spatial autocorrelation. The value of this analysis is particularly great when it is assumed that some non-Poisson point process is responsible for the observed spatial pattern. Thus, a clustered pattern may itself be considered a null hypothesis that can be tested for further clustering. In addition, a number of patterns in the same area representing different variables may be compared. See Bailey and Gatrell (1995), and Getis and Franklin (1987).

*Spatial autocorrelation coefficients.* In regression models where estimation is based on georeferenced data, it is mandatory that any statistically significant spatial effect must be accounted for in the model. The spatial effects can be diagnosed by means of Moran's $I$ tests on residuals or on variables that are to be included in the model. Also, regardless of diagnostics, spatial dependencies may be subsumed by creating spatial autoregressive models of one kind or another. Two popular autoregressive models are (i) the mixed regressive spatial autoregressive model, often called the spatial lag model,

$$\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{B.3.13}$$

and (ii) the linear regression with a spatial autoregressive error, or simultaneous autoregressive model (SAR), often called the spatial error model

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + (\boldsymbol{I} - \lambda \boldsymbol{W})^{-1}\boldsymbol{\mu}. \tag{B.3.14}$$

In both of these cases, parameters representing spatial effects, $\rho$ and $\lambda$ must be determined. Note that in each case they precede the $\boldsymbol{W}$ matrix, which takes any of the forms discussed above. In essence, the coefficients reveal the strength or influence of the $\boldsymbol{W}$ matrix. In so doing, they become spatial autocorrelation coefficients; high positive or negative values represent strong spatial effects and low values the opposite. When $\rho, \lambda$ are zero, there are no spatial effects. This is true since the error terms $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ respectively are randomly distributed in space. If, in estimation of the models, errors are spatially correlated, the models are misspecified. In addition to Moran's *I* regression residual test, specialized tests such as the the Kelejian and Robinson (KR) test (1993), or the Wald, Likelihood Ratio, and Lagrange multiplier tests are used to identify spatial autocorrelation in spatial lag or spatial error type models (Anselin 2006). For example, for the KR test, normality of errors is not required, nor is it necessary to hypothesize a strictly linear model. In addition, KR studies only certain selected contiguity relationships (Kelejian and Robinson 1993). For details on spatial autocorrelation coefficients, see Anselin (1988). Anselin (2006) presents a comprehensive review of spatial econometrics.

### *Local measures and tests*

Among spatial analysts, there has always been an interest in focused measures, that is, a desire to describe precisely the 'situation' or proximity characteristics of a particular site. But it was not until the invention of local statistics that it became possible to measure and test for certain situational characteristics. What better way is there to investigate situational characteristics of sites than to use spatial autocorrelation measures and tests? The basis for local tests for and measures of spatial autocorrelation comes from the cross-product statistic. This time the structural form is

$$\Gamma_i = \sum_{j=1}^{n} W_{ij} Y_{ij} \qquad i \neq j. \tag{B.3.15}$$

Note that here we are finding the interaction between spatial weights in the *i*th vector only and the *y* values in *Y*'s *i*th vector. $\Gamma_i$ allows for autocorrelative comparisons between the two vectors for a given site *i*.

*Getis and Ord local statistics.* These statistics are additive in that the focus is on the sum of the *j* values in the vicinity of *i*. The fact that there are two statistics, $G_i$ and $G_i^*$, allows researchers to choose hypotheses based on proximity ($G_i$) or on clustering ($G_i^*$). $G_i^*$ is written as

$$G_i^*(d) = \frac{\sum_{j=1}^{n} W_{ij}(d) y_j - W_i^* \bar{y}}{s\{[(nS_{1i}^*) - W_i^{*2}]/(n-1)\}^{1/2}} \qquad \text{for all } j \qquad \text{(B.3.16a)}$$

where

$$W_i^* = W_i + W_{ii} \quad \text{and} \quad S_{1i}^* = \sum_{j=1}^{n} W_{ij}^2 \qquad \text{for all } j \qquad \text{(B.3.16b)}$$

and $\bar{y}$ and $s$ are the mean and standard deviation, respectively.

The mathematical distinction between the two statistics depends on the role of the *i*th observation. If our concern is with the effect of the influence of *i* on the *j* values, the focus is on the site *i* but not the *y*-value associated with it. Thus, the view is one of proximity (situation). The null hypothesis would be: there is no association between *i* and its neighbors *j* up to distance *d*. The $G_i^*$ statistic, on the other hand, includes the value $y_i$ in its calculations; it sums associations between *i* and *j* including *i* (the value for $W_{ii}$ – usually one – is added to $W_i$). Thus, $G_i^*$ lends itself to studies of clustering since a cluster usually contains its focus as a member of the cluster.

Both statistics are distributed normally. They are scaled in such a way that $G_i(d)$ and $G_i^*(d)$ are equivalent to standard deviations of the normal distribution. Thus, there is no need to convert the statistics. It is interesting to note that $G_i^*$ is mathematically associated with global Moran's $I(d)$ so that Moran's $I$ may be interpreted as a weighted average of the local statistics (Getis and Ord 1992; Ord and Getis 1995). For these statistics as well as all other spatial autocorrelation statistics, boundary effects may lessen the number of associations between *i* and *j*. To avoid the resulting bias, boundary effects should be minimized by judicially selecting the area of study. Hot spots identified by these statistics can be interpreted as clusters or indications of spatial nonstationarity.

*Local indicators of spatial association – LISA.* LISA statistics were created by Anselin (1995), whose motivation was to decompose global statistics such as Moran's $I$ and Geary's $c$ into their local components for the purpose of identifying influential observations and outliers. The individual components of $I_i$ are related

to $I$. Just as the $\Gamma_i$ sum to $\Gamma$, so too will all $I_i$ sum to $I$, subject to a factor of proportionality. Local Moran's $I_i$ is defined as

$$I_i = \frac{y_i - \bar{y}}{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2} \sum_{j=1}^{n} W_{ij}(y_i - y_j) \qquad i \neq j, \text{ for } j \text{ within } d \text{ of } i \qquad \text{(B.3.17)}$$

and the factor of proportionality is

$$\gamma = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} \sum_{i=1}^{n}(y_i - \bar{y})^2 . \qquad \text{(B.3.18)}$$

The expected value of

$$E(I_i) = -\frac{1}{n-1}\sum_{j=1}^{n} W_{ij} . \qquad \text{(B.3.19)}$$

Tests for spatial autocorrelation may be carried out either using the moments of the $I_i$ distribution (see Anselin 1995) or by random permutations. The second technique, the strategy of conditional randomization, is preferred for LISA since the possible existence of global autocorrelation would otherwise affect the interpretation of $I_i$ (Anselin 1995).

For Geary's $c$, the local version is

$$c_i = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2} \sum_{j=1}^{n} W_{ij}\left[(y_i - \bar{y}) - (y_j - \bar{y})\right]^2 . \qquad \text{(B.3.20)}$$

Here the factor of proportionality is

$$\gamma = \frac{2n}{(n-1)} \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij} . \qquad \text{(B.3.21)}$$

The LISA statistics are particularly useful for identifying spatial clusters. High spatial autocorrelation values indicate clusters of high or low values. Software provided in GeoDa (discussed below and in an earlier section of this book) provides graphics in which the ++, – –, + –, and – + types of spatial association are differentiated. Sokal et al. (1998) take a different view of local analysis, and Boots (2002) analyzes local measures of spatial autocorrelation.

*Geographically weighted regression.* A local version of an ordinary least squares regression analysis has been proposed by Fotheringham et al. (1995). The point of geographically weighted regression (GWR) is that regression parameters are not constant over space as characterized by traditional regression models and that the variation can be explicitly modeled. By using a $\boldsymbol{W}$, usually a Gaussian or near-Gaussian spatial weights decline function for each $i$ as elements in the matrix, a regression can be estimated for each $i$th location. Although each weight matrix need not be focused on data sites, the point of the analysis is to estimate the variation in parameters across space. The form of GWR can be written as

$$\boldsymbol{Y} = (\boldsymbol{\beta} \otimes \boldsymbol{X})\, \boldsymbol{1} + \boldsymbol{\varepsilon} \tag{B.3.22}$$

where the logical operator (Kronecker product) $\otimes$ requires that corresponding elements in each matrix are multiplied by each other. Since each matrix has $n$-*by*-$(k+1)$ dimensions, where the number of independent variables is $k$, the vector of ones with dimensions $(k+1)$-*by*-1 yields the required $n$-by-1 matrix for $\boldsymbol{Y}$. This allows $\boldsymbol{\beta}$ to consist of $n$ sets of local parameters. Each set contains a slope and intercept for each independent variable for each $i$. The *betas* are estimated by use of a $\boldsymbol{W}$ for each $i$. The $d$ for all $\boldsymbol{W}$s is either selected in advance or estimated from the data. A typical $\boldsymbol{W}$ is based on a pre-selected outer distance bandwidth $b$

$$W_{ij} = \begin{cases} \left[1 - \left(\dfrac{d_{ij}}{b}\right)^2\right]^2 & \text{if } d_{ij} < b \\ 0 & \text{otherwise.} \end{cases} \tag{B.3.23}$$

Often, in a single study, $b$ is allowed to vary because standard errors might be particularly high when the $b$-radius includes only a few data points around $i$. Various systems are provided for selecting $b$ (Fotheringham et al. 2002). A result of GWR is a map of what might be called 'parameter space.' Areas with high parameter values indicate particularly strong correlative relationships between regressor and response variables, but the parameters are not directly indicative of spatial autocorrelation. Since the beta values are a function of the spatial weighting scheme, to the extent that $\boldsymbol{W}$ captures the spatial autocorrelation effects in each of the vari-

ables, it is reasonable to say that high beta values reflect on the pattern of spatial autocorrelation in the system. It is possible, however, to specify autoregressive instead of OLS models, thus the GWR parameters can play the same role as in spatial autoregressive models. The implication is that one or more spatial autocorrelation maps can be produced for each equation in the system. Fotheringham, Brunsdon, and Charlton continue to write many articles on this subject. Reviews and analyses are found in Páez et al. (2002), Leung (2000) and Wheeler and Tiefelsdorf (2005) and in Chapter C.6 of this Handbook.

*Local spatial autocorrelation in the presence of global spatial autocorrelation.* As mentioned in our necessarily short discussion of local statistics, when global spatial autocorrelation exists, it becomes difficult to interpret the nature of local spatial autocorrelation. How much of a statistically significant result for a local test on $i$ is due to pervasive global autocorrelation? It may be that local statistical significance is just an artifact of the larger scale effect due to global associations. Ord and Getis (2001) provide a test, called $O$, that includes separate information on the observations within $d$ of $i$ (regular or irregular areas) representing the hypothesized hot spot, and on observations immediately outside of the hot spot. The statistic is

$$O_i(d) = \overline{Y}_d - \overline{Y}_0 \tag{B.3.24}$$

where $\overline{Y}_d$ is the mean of the $n(d)$ observations within $d$ and $\overline{Y}_0$ is the mean of the $m = M - n(d)$ observations, the $M$ being a regionally partitioned set of observations that displays 'relative homogeneity.' The $M$ should be considerably larger than $n(d)$ [at least 10 times] but considerably smaller than all $n$ observations in the study area. $M$ can be selected to include all observations from $i$ [(except the $n(d)$] up to the *range* (in the geostatistical sense) derived from all observations. The idea of the statistic is to compare characteristics of data at two spatial scales; $E[O_i(d)] = 0$. Testing procedures are given in Ord and Getis (2001). Boots and Tiefelsdorf (2000) consider the relationship of global to local measures of spatial autocorrelation.

## B.3.5 Problems in dealing with spatial autocorrelation

It is clear that spatial autocorrelation can be defined precisely, but it is not always clear whether the various measures and tests just described can actually find spatial autocorrelation in georeferenced data. Each of them has its own shortcomings, but more important, they perform better or worse depending on the way in which $\boldsymbol{W}$ and $\boldsymbol{Y}$ are specified. For example, results depend on the nature of $\boldsymbol{W}$, again emphasizing the importance of a meaningful specification of $\boldsymbol{W}$. Much work remains to be done to better understand the effects of various $\boldsymbol{W}$ matrices on results. Simi-

larly, $\boldsymbol{Y}$ will yield different results depending on the nature of the associations specified for the realizations of $\boldsymbol{Y}$. The fundamental question for researchers in this area is: What is responsible for any spatial autocorrelation that exists in a particular data set? Is it the way the boundaries of the spatial units were drawn (the geometry and/or scale of the spatial units under study) or is it a function of the nature of the variables under study? When spatial autocorrelation is embedded within a variable, is it because of the geometry of the spatial units or something else? A number of commentators have discussed the problems in dealing with spatial autocorrelation including Bao and Henry (1996), Legendre (1993), and Pace and Barry (1997).

A particularly difficult area of research is the selection of tests that can withstand the simultaneity effects of multiple tests. Especially in local statistics, usually there are tests on spatial autocorrelation for each data site. This results in very large numbers of tests that are in fact dependent on one another. Thus we come to the ironic situation where in the search for spatial autocorrelation we are subject to the effects of spatial autocorrelation itself. For example, many of the observations used to find a local measure of spatial autocorrelation will be used again for a test focused on a neighboring observation. There have been several attempts to resolve this problem of simultaneous, dependent tests (Getis and Ord 2000; Castro and Singer 2006; Benjamini and Hochberg 1995). Also see Chapter B.4. Researchers must be conscious of Bonferroni-type confidence intervals when they select their diagnostic and testing devices.

Many traditional tests require the assumption of stationarity. Checking for stationarity in empirical work is a good practice. GWR, while attempting to get around this problem, falls prey to problems of sample size (necessarily small for estimates of $Y_i$ ) and to the overlapping test problem.

The problem of the effect of global spatial autocorrelation on local effects was alluded to above. Is that relationship fully understood? What about the effect of boundaries on levels of confidence? Sample size and thus the number of degrees of freedom are affected by the spatial extent of study regions. For example, does the distance $d$ include suitable numbers of observations that allow for acceptable levels of confidence in results? How is $d$ to be selected? Careful attention must be given to the effect of various $d$ on results. A promising technique of analysis, spatial filtering, may be particularly useful in answering many of these questions (Getis 1990, 1995; Griffith 1996, 2003; Griffith 2002). See also Chapter B.5 in this volume. Many of these problems can be better understood in a framework of exploratory spatial data analysis. The software packages mentioned in the next session are designed to assist in exploration and model development and testing.

### B.3.6 Spatial autocorrelation software

Tests and measures of spatial autocorrelation are available in a number of software packages. Most often in these packages, finding and testing for spatial autocorrelation is only one part of a large variety of spatial analytic procedures.

*GeoDa*. Perhaps the most comprehensive package is GeoDa (Anselin et al. 2006), which provides a number of exploratory procedures that elicit information about spatial patterns. In addition, tests and analysis of spatial autocorrelation are available in a number of different segments of the software, including the estimation and testing of a variety of spatial econometric models. Novel graphical and mapping procedures allow for detailed study of global and local spatial autocorrelation results. Non-stationarity and outliers can be assessed by means of maps of statistically significant clusters. See Chapter A.4 of this volume for further explanations.

R *Packages.* Two noteworthy packages are based on the R language environment. One is the *spdep*, a package with many spatial data exploratory functions, graphics, and hypothesis tests on spatial autocorrelation (Bivand 2006). A package specifically designed for the study of point pattern processes is *Spatstat* (Baddeley and Turner 2005). A special feature of this package is simulation routines for different types of point pattern processes. Tests and diagnostics are included. See Chapter A.3 for a fuller treatment of this package.

*PPA* (*Point Pattern Analysis*). This small package includes routines for global and local spatial autocorrelation statistics. Included are nearest neighbor and K function procedures and tests. Graphics are not included (see Aldstadt et al. 2002).

*SANET* is a toolbox that allows for the study of spatial autocorrelation on networks (Okabe et al. 2006).

*STARS* (*Space-Time Analysis of Regional Systems*) is an exploratory package that brings together a number of recently developed methods of space-time analysis into a graphical environment. Spatial autocorrelation can be studied on dynamically-viewed time-dependent maps. Many descriptive statistics are available, as in *GeoDa*, that are keyed directly to individual observations on maps (Rey and Janikas 2006). See Chapter A.5.

*ArcGIS.* This large system of spatial data management and analysis contains modules that allow for map study with *K* functions and autocorrelation statistics (ArcVIEW 9.3). More detail is available in Chapter A.1. Recent versions contain routines for GWR. One module, *Geostatistical Analyst*, provides a large number of descriptive and analytical routines for the study of semivariograms (ESRI 2001).

*ClusterSeer 2*. Developed primarily for health science spatial research, this package makes available a number of pattern analytic routines popular in disease and crime research. The routines identify statistically significant spatial clusters whether or not the focus is on a particular observation or a particular site. The concept of spatial autocorrelation is embedded in many of the routines (Jacquez et al. 2002).

*Le Sage's Spatial Econometrics Toolbox.* This package contains an extensive collection of MATLAB econometric functions, many of which were created for spatial data (LeSage 1999, 2004).

*Spatial Statistics and SAS*. By means of SAS procedures, Griffith (see Chapter A.2 of this volume) has created specialized routines that allow for the analysis of spatial econometric systems, in particular, spatial filtering (see Chapter B.5 of this volume).

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